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THEORY OF PION SCATTERING BY NUCLEI II

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ABSTRACT

The Chew-Low theory is used to calculate the scattering of a pion by a nucleon inside nuclear matter. Together with the standard relation between refractive index and forward scattering, this yields the wave number of a pion inside nuclear matter. Singularities appearing in earlier theories are avoided.

We have obtained what seems to us a satisfactory solution for the scattering of pions by nuclear matter. Some of the reasons for the search for a new theory of this phenomenon are given in Ref. 1.

One part of the theory is the general relation for the refractive index

$$k^2 - k_0^2 = 4\pi\rho f(k,0)\,, \qquad (1)$$

where k is the wave number inside nuclear matter, k_0 is that for free pions,

$$k_0^2 = \omega^2 - \mu^2\,, \qquad (2)$$

ρ is the number of scatterers (nucleons) per unit volume, and f(k,0) is the amplitude of forward scattering ($\theta = 0$) of a pion by a single nucleon inside nuclear matter. The amplitude f is related to the scattering matrix by

$$f(k,0) = -(\omega_k/2\pi)\,\overline{\langle k,\omega|T|k,\omega\rangle}\ . \qquad (3)$$

The bar above the matrix element of the T-matrix indicates an average over nuclear states. The factor $-(\omega_k/2\pi)$ is appropriate to the definition of the T-matrix $t_{qp}(z)$ of Chew and Low[2] (quoted as CL), their Eq. (32).

The main part of the theory is concerned with the calculation of T inside nuclear matter. CL have shown that the scattered amplitude consists of a "trivial" factor depending on k (here the factor is k^2) and an angle [CL Eq. (33)], and an intricate factor depending on ω. To obtain the latter, we use the CL dispersion relation approach, which may be interpreted in terms of the Feynman diagram (Fig. 1) for 33-scattering (read from right to left). In the intermediate states, *a* to *e*, at least one "intermediate" pion, $\vec{k}_1$ and/or $\vec{k}_2$, is present. CL have shown that the resonance in the 33 states arises chiefly from intermediate states of large pion momentum. This has two consequences:

(1) Because, for large momentum, the relation between pion momentum and energy should be the same in nuclear matter as in space, the high-momentum intermediate states should give the same contribution in the dispersion integral as for CL.

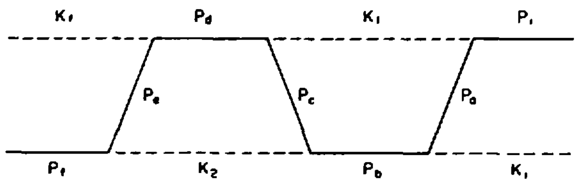

Fig. 1.
Feynman diagram of scattering of pion (dashed) by nucleon (solid). Momentum is conserved and is equal to $\vec{p}_i + \vec{k}_i$ at every stage.

(2) If the pion momenta $\vec{k}_1, \vec{k}_2$ are large, the nuclear momenta $\vec{p}_a ... \vec{p}_e$ are large also. Therefore, there should *not* be any appreciable effect of the *Pauli principle* in *intermediate* states, in contrast to the claim made in Ref. 1.*

Item 1 can be used to calculate the K-matrix immediately. To evaluate the K-matrix it is necessary to perform a dispersion integral of the form

$$P \int \frac{F(\omega')}{\omega' - \omega} d\omega' \ . \tag{4}$$

Because of the principal value operator P, intermediate states ω' near ω do not give a very important contribution. Therefore, the K-matrix should have essentially the same value in nuclear matter as for CL, i.e.,

$$< \omega | K | \omega > \sim \frac{1}{1 - \omega/\omega_r} \ , \tag{5}$$

where ω_r is the usual resonance energy in the 33 states,[3]

$$\omega_r = 2.14\mu \ . \tag{6}$$

To obtain the T-matrix, unitarity must be incorporated. It is not obvious how to do this when k is complex. After many futile attempts, we used the CL theory once more. In this theory, the complex ω-plane is considered. This makes unitarity much easier to apply, because we may deform the path of integration so that it runs along the line of real k, which of course makes ω complex. Real momentum allows us to define a phase shift δ_α, which is taken to be a real number because probability is conserved when a pion is scattered from a nucleon in matter. If k is real, and if we neglect the Pauli principle (see below), the quantity $h_\alpha(\omega)$, defined by CL Eq. (32), is the same for the nuclear matter problem as CL Eq. (34), namely,

$$\lim_{z \to \omega_k + i\epsilon} h_\alpha(z) = e^{i\delta_\alpha} \sin [\delta_\alpha(k)] / k^3 v^2(k) \ , \tag{7}$$

where the factor k^3 comes from the use of p-state mesons, and v(k) is a form factor of the nucleon, CL Eq. (4). Introducing $g_\alpha(z)$ by CL Eq. (41),** we then use CL Eq. (42) to determine the discontinuity* of this quantity across the line of real k.**

$$\lim_{z \to \omega_k + i\epsilon} g_\alpha(z) - \lim_{z \to \omega_k - i\epsilon} g_\alpha(z) = -2i\lambda_\alpha k^3 \mu^{-2} \omega_k^{-1} v^2(k) \ . \tag{8}$$

The only difference is that ω_k is now complex. The behavior of $g_\alpha(z)$ is therefore the same as in CL Eq. (45), and F_α has the same value as CL Eq. (46), namely,

$$F_\alpha(\omega_k) = \lambda_\alpha k^3 \mu^{-2} \omega_k^{-2} v^2(k) \ . \tag{9}$$

The "effective range approximation," Sec. V. of CL, can be made. Using this and Eq. (8), we get along the line of real k

$$\lim_{z \to \omega_k + i\epsilon} g_\alpha(z) = 1 - \omega_k/\omega_r - i\lambda_\alpha k^3 \mu^{-2} \omega_k^{-1} v^2(k) \tag{10}$$

This, being an analytic function, can now be used equally well along the (physical) line of real ω, which can be reached without crossing any branch cut. This yields,† for moderate k where v(k) = 1,

$$g_\alpha(\omega) = 1 - \omega/\omega_r - i\lambda_\alpha k^3/\mu^2 \omega \ . \tag{11}$$

Equation (11) can be re-inserted into CL Eqs. (41), (32), and (33), and gives for the forward scattering amplitude, after some algebra,

$$t_{kk}(z) = -2C_3 \, k^2 \, \frac{2\pi}{\omega_k} \, \frac{\lambda_3}{\mu^2 z g_3(z)} \tag{12}$$

where

$$\begin{aligned} C_3 &= 1 \text{ for } P\pi^+ \text{ or } N\pi^- \ , \\ &\quad 1/3 \text{ for } P\pi^- \text{ or } N\pi^+, \text{ and} \\ &\quad 2/3 \text{ for } P\pi^0 \text{ or } N\pi^0 \ . \end{aligned} \tag{13}$$

For equal numbers of neutrons and protons (ordinary nuclear matter), $C_3 = 2/3$ regardless of the charge of the pion.

*J. Negele and G. Baym, independently, pointed out to us that this claim was false.

**We are grateful to Drs. Boulware and Blaha and to other members of the theoretical physics seminar of the University of Washington for suggestions leading to this choice.

*Strictly speaking, because ω_k is complex, we should not look for the discontinuity of g_α, but for the value of its imaginary part for $\omega_k + i\epsilon$. We are convinced that the result in Eq. (10) remains unchanged as long as Im ω_k is small.

**CL use $\mu = 1$. To make g dimensionless, we must introduce the factor μ^{-2}.

†That the imaginary part is proportional to k^3, with k the meson wave number *in* the medium, was found previously by S. Barshay et al.[4]

Collecting all formulae, we obtain

$$y^2 = x^2 - 1 + \frac{BCr\,y^2}{x(a-x) - iBy^3}\ , \tag{14}$$

where

$$x = \omega/\mu,\quad y = k/\mu,\quad a = \omega_r/\mu,$$

$$B = \frac{4}{3} f^2 \frac{\omega_r}{\mu}\ ,\ C = \frac{8\pi\rho_0}{\mu^3} C_3,\ \text{and}\ r = \frac{\rho}{\rho_0}\ . \tag{15}$$

Using $C_3 = 2/3$ and $f^2 = 1/4\pi \approx 0.080$, Eq. (14) was evaluated on the computer. These results are shown in Figs. 2 and 3. In Fig. 2, k/μ is plotted against k_0/μ for a *fixed* density, namely, normal nuclear density ρ_0. In Fig. 3, k/μ for a fixed, low pion energy (kinetic energy 36.0 MeV) is plotted against the density.

Some of the features of the solution are as follows.

1. The real part of k is usually considerably larger then the imaginary part.
2. k never gets excessively large.
3. k is not a simple function of either k_0 or ρ.
4. Above a certain critical density ρ_1, the value of k for $k_0 = 0$ is nonvanishing. For ordinary nuclear matter (equal numbers of neutrons and protons),

$$\rho_1 = \frac{9}{8}\frac{\mu^3}{8\pi f^2}\left(1 - \frac{\mu}{\omega_r}\right) = 0.103\ \text{fm}^{-3}\ . \tag{16}$$

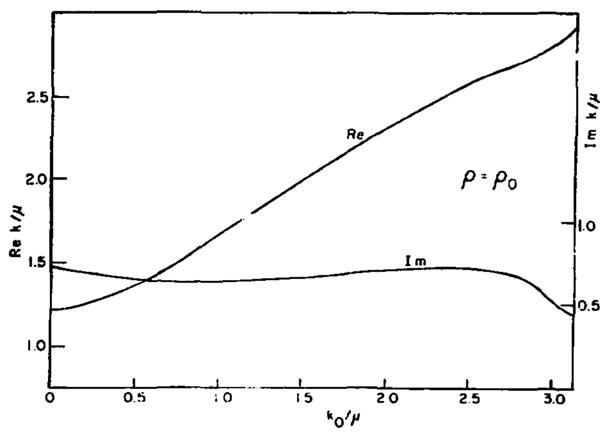

Fig. 2.
Real and imaginary part of the wave number k in nuclear matter vs k_0, the wave number of a free pion. Density ρ_0 = normal nuclear matter density (0.16 fm^{-3}).

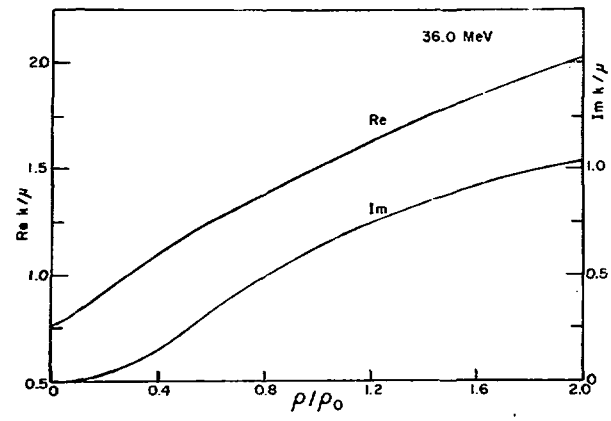

Fig. 3.
Wave number k at pion kinetic energy 36.0 MeV vs nuclear matter density ρ/ρ_0 (ρ_0 = 0.16 fm^{-3}).

The density of normal nuclear matter is

$$\rho_0 = 0.16\ \text{fm}^{-3} = 1.5\,\rho_1\ .$$

The present theory must be extended in many ways, some of which are as follows.

1. Pauli principle: The imaginary part of g in Eq. (10) represents the attenuation of the main wave due to actual scattering, which in nuclear matter means quasi-elastic scattering. In the *final* state of this type of scattering, the Pauli principle *will* operate. We were able to calculate the effect of this for real k. Unfortunately, this is not an analytic function of k, thus it can be extended only approximately to complex k.
2. Finite nuclei: Here we propose to use the "Schrödinger equation"

$$\nabla^2\psi + k^2(\rho,\omega)\psi = 0\ , \tag{17}$$

where $k(\rho,\omega)$ is the function calculated for nuclear matter from Eq. (14). It would be desirable to replace Eq. (17) by a self-adjoint equation.

3. Other partial waves: The calculation of the contribution of the 11, 13, and 31 partial waves, as well as the s-wave, should be straightforward because the phase shifts are small.
4. The Lorentz-Lorenz correction should be included.

We hope to make these modifications in a subsequent report, to make comparisons with experiment and with previous theories.

For an experimental test of the theory, the quasi-elastic scattering into the backward hemisphere will probably be best. The fact that $k \gg k_o$ for the pions will greatly increase the momentum transfer to the recoil nucleon compared to free pion collisions. Sometimes the pion may lose so much energy that it gets stuck in the nucleus, and enhanced absorption γ rays may be observed.

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3. For the mass of Δ, 1236 MeV, see Barash-Schmidt et al., "Tables of Particle Properties," Rev. Mod. Phys. 45, S25 (1973).

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